Using micro-perforated plates to realize a silencer based on the Cremer impedance

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Summary
Current trends for IC-engines are driving the development of more efficient engines with higher specific power. This is true for both light and heavy duty vehicles and has led to an increased use of charging. The charging can be both in the form of a single or multi-stage turbo-charger driven by exhaust gases or via a directly driven compressor. In both cases a possible noise problem can be a strong Blade Passing Frequency (BPF) typically in the kHz range and above the plane wave range. In this paper a novel type of compact dissipative silencer developed especially to handle this type of problem is described. The silencer is based on a combination of a micro-perforated tube backed by a locally reacting cavity. The combined impedance of micro-perforate and cavity is chosen to match the theoretical optimum known as the Cremer impedance at the mid-frequency in the frequency range of interest. Due to the high damping achieved at the Cremer optimum (hundreds of dB/m) it is easy to create a compact silencer with a significant damping (say >40 dB) in a range larger than an octave. Several principles are presented to determine the parameters of micro-perforate and cavity. The numerical results indicate that, following the principles, a silencer with broad-band damping can be achieved.

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1. Introduction
A micro-perforated plate (MPP) consists of a plate with small perforations distributed over its surface. When these perforations are of sub-millimeter size they provide by themselves enough acoustic resistance and low acoustic mass reactance necessary for a wideband absorber[1]. Also MPP:s made of metallic plates are robust and can be cleaned more easily to maintain their damping ability. In order to apply MPP:s for applications outside the traditional room acoustic area knowledge about the effect of high sound levels and flow speed are important. Due to the fact that MPP:s have small perforation ratios typically around 1%, a large increase (~100) in the acoustic velocity in the holes or slits occurs. This leads to acoustically induced vortex shedding already at moderate incident sound pressure levels (~90 dB). This effect is actually included in the original papers by Maa[1]. The existing published MPP impedance formulas have been summarized by Guo et. al.[2] who also added the effects of flow. The goal of the present paper is to extend the work of Allam and Åbom[3],[4] in which a novel type of MPP silencer was described. A specific impedance boundary condition is necessary to optimally suppress the noise at different conditions. In this paper optimization of the MPP silencer is studied based on trying to create a wall impedance matching the so called Cremer impedance[5]. The work of Cremer was revisited by Tester[6],[7] who also included mean flow in the analysis. The procedure to find the optimal impedance of a lined duct carrying a mean flow is first summarized and the work of Tester[7] extended to include cases close to cut-on. One limitation in the work of Tester[7] is that the solution for the optimum impedance only is studied for frequencies well beyond the plane wave range. The motivation being that this is the range of interest for aero-engines. Here this assumption is removed and it is demonstrated that for low frequencies, i.e., below or around the first mode cut-on, the optimum impedance can differ significantly from the high frequency case. For applications around the plane wave range, e.g., IC-engines, this is important.
since the correct optimum results in much higher damping values.

In order to estimate the performance of the optimized silencer, it is necessary to predict the sound propagation in a lined duct. As we know, if the eigenvalue of the softwall has been calculated, the mode-matching method [8] can be used to predict the attenuation of multi-segmented liners. The eigenvalue equation derived from the impedance condition is usually solved by Eversman’s integration method[9],[10], in which a nonlinear ordinary differential equation (ODE) is derived by introducing a parameter perturbation to the transcendental algebraic equation. The solution of eigenvalues is thus changed into an initial-valued problem. With this method, one can obtain the eigenvalues by numerically solving the ODE with the hard wall eigenvalues as the initial condition of the soft wall. Sun et al[11] presented a unified algorithm to study the eigenvalue problem for a lined duct using the homotopy method. The basic concept of the homotopy method is used to constitute various homotopy equations to calculate the eigenvalues for both locally and non-locally reacting liners. In this method, the homotopy equation suggested has no singularity for the lowest mode, i.e., the plane wave mode, and one can flexibly choose homotopy parameters to avoid numerical instability or mode jump problems. In the present paper, the eigenvalues were solved with the algorithm presented by Sun et al[11] and mode-matching method[8] is used to compute the performance of the optimized silencer.

2. The silencer studied and the Cremer impedance

Acoustic liners and perforated mufflers are used to suppress noise in turbofan engine ducts and internal combustion engine (ICE) exhaust systems, respectively, by providing appropriate impedance boundary conditions. The configuration of the MPP silencer studied here is given in Figure 1. As shown in Ref. [3] the original expansion chamber behavior is suppressed by inserting rigid walls and sub-dividing the outer chamber. For equally long divisions one more minimum (at a multiple of ) in the expansion chamber transmission loss can be suppressed. In this paper the locally reacting limit, i.e., increasing the number of chambers so that their length is always less than in the range of interest will be investigated. To analyze the proposed idea a compact prototype silencer with the geometry shown in Figure 1 will be used. The chosen dimensions are typical for what would be expected for an automotive application, e.g., control of a compressor BPF in the kHz range.

![Figure 1. Cross-section of the proposed dissipative MPP silencer[3] for noise control in e.g. automotive intake/exhaust systems. The geometry is assumed to be circular symmetric and the outer volume (cavity) is sub-divided in order to create a locally reacting response.](image1)

![Figure 2. Sound propagation in a duct with a locally reacting surface with impedance Z. (a) infinite liner; (b) finite liner (the walls outside the liner are assumed rigid). For the infinite liner a particular mode is assumed, but for finite liner the transmitted and reflected waves can also contain other modes.](image2)
\[ TL = 10 \log_{10} \frac{W_{in}}{W_{out}}, \] (1)

\( W_{in} \) is the acoustic power determined by the amplitude of incident sound wave \( p_{+1} \), as shown in Figure 2. The primary problem for the design of a locally reacting silencer is to find the optimal impedance resulting in a maximum transmission loss for a given frequency. The solution will depend on the mode studied. Here the solution for the zero order or plane wave mode, which is of interest for many applications, e.g., automotive, will be at focus. In 1953, Cremer[5] derived analytic expression for the optimal normalized impedance for the zero mode of an infinitely long liner covering one face of a uniform rectangular duct. It is given by

\[ Z_{\text{Cremer}}|_{M_{s}=0} = (0.91 - 0.76j) \frac{k h}{\pi}, \] (2)

where \( j \) is the imaginary unit, \( k \) denotes the wave number and \( h \) is the height of the duct perpendicular to the liner. \( M_{s} \) is the Mach number of mean flow. Tester[6] later made a modification to equation (2) for the rectangular duct problem giving the formula

\[ Z_{opt}|_{M_{s}=0} = (0.929 - 0.744j) \frac{k h}{\pi}. \] (3)

The concept of Cremer impedance was also extended to circular ducts by Tester[6] and the optimal impedance for the zero mode or plane wave is

\[ Z_{opt}|_{M_{s}=0} = (0.88 - 0.38j) \frac{k r_{0}}{\pi}, \] (4)

where \( r_{0} \) is the radius of the duct. Thus, in the absence of mean flow, equation (4) can be used to calculate the optimal impedance of a silencer for an infinitely long liner, as shown in Figure 2(a). For both turbofan and IC-engines applications it is necessary to include mean flow and Tester[7] extended Cremer’s result[5] to ducts containing uniform flow. Following Tester’s study[7] for rectangular ducts it is here suggested that, in a circular duct, the radial eigenvalues \( k_{r} r_{0} \) creating optimal damping for modes with circumferential order \( m \) are the solutions of the equation

\[ \frac{d}{d(k_{r} r_{0})} \left[ \frac{(1 - k_{m} M_{s})^{2}}{k} J_{m}(k_{r} r_{0}) \right] = 0. \] (5)

where \( J_{m} \) is the Bessel function of first kind of \( m \) th order. Equation (5) is based on the observation by Cremer[5] and generalized by Tester[6], that the optimum damping for a mode pair occurs at a branch point of the eigenvalue equation \( f(k_{r}) = 0 \). At this branch point defined by \( df/d(k_{r}) = 0 \), two modes merge. The wave number in the axial direction is

\[ k_{x} = \frac{M_{s}}{k} \pm \sqrt{1 - (1 - M_{s}^{2})(k_{r}/k)^{2}}, \] (6)

For \( m = 0 \), or zero mode, equation (5) can be written as

\[ \frac{d}{d(k_{r} r_{0})} \left[ \left(1 - k_{m} M_{s} \right)^{2} k_{r} J_{m}(k_{r} r_{0}) \right] / J_{m}(k_{r} r_{0}) = 0. \] (7)

For a “well-cut-on” incident sound wave, equations (6) and (7) are, approximately,

\[ k_{x} = \frac{k}{1 + M_{s}}, \] (8)

\[ \frac{d}{d(k_{r} r_{0})} \left[ \left(1 + M_{s} \right)^{2} k_{r} J_{m}(k_{r} r_{0}) \right] / J_{m}(k_{r} r_{0}) = 0. \] (9)

The optimal non-dimensional radial wave numbers for the “well-cut-on” case are therefore independent of the flow Mach number and wave number \( k r_{0} \). The \( (k_{r} r_{0})_{opt}|_{M_{s}} \) eigenvalue for \( m = 0 \) is under this assumption identical to the zero flow value of

\[ (k_{r} r_{0})_{opt}|_{M_{s=0}} \approx (k_{r} r_{0})_{opt}|_{M_{s=0}} = 2.98 + 1.28j. \] (10)

Thus, the optimal impedance for zero mode in circular ducts containing uniform flow is given by the simple relation[7]

\[ Z_{opt}|_{M_{s}} = \frac{Z_{opt}|_{M_{s=0}}}{\left(1 + M_{s} \right)^{2}} \sim \frac{0.88 - j0.38}{\pi} k r_{0}. \] (11)

It is true that equation (11) gives the approximate solutions of optimal impedance under “well-cut-on” situation. The solutions of equation (7), however, are very different from the solutions of equation (9) in the range below or around cut on, which is addressed in the present study. The non-dimensional optimal radial wave number depends on the flow Mach number \( M_{s} \) and wave number \( k r_{0} \). Let

\[ f_{1} = 1 - M_{s} \frac{k}{k}, \] (12)

after some manipulation, equation (7) is
The optimal radial wave number can be obtained by solving equation (14). The optimal radial eigenvalue \((k_r \rho_1)_{opt}\) is dependent of the non-dimensional wave number \(k_r \rho_1\) for a particular flow Mach number. Table 1 shows the first branch solutions of equation (14) for different wave number \(k_r \rho_1\) and flow Mach number. The Mach numbers used are typical for an automotive intake system application. The results indicate that the optimal radial wave number \((k_r \rho_1)_{opt}|_{\text{lin}}\) are very different from the solution of zero flow in equation (10). The difference increases with the flow Mach number and decreases with increasing dimensionless wave number. The solutions of equation (14) are identical to equation (9) for \((k_r \rho_1) = 16\), which represents the sound wave in the “well-cut-on” region when equation (11) can be used to calculate the optimal impedance.

| \(k_r \rho_1\) | \((k_r \rho_1)_{opt}|_{\text{lin}}\) |
|---|---|
| \(M_t = 0.05\) | \(M_t = 0.10\) | \(M_t = 0.15\) |
| 0.95 | 3.15+1.34j | 3.31+1.39j | 3.46+1.43j |
| 1.11 | 3.12+1.33j | 3.27+1.38j | 3.40+1.41j |
| 1.84 | 3.07+1.30j | 3.17+1.32j | 3.25+1.34j |
| 3 | 3.04+1.27j | 3.10+1.26j | 3.15+1.25j |
| 16 | 2.98+1.28j | 2.98+1.28j | 2.98+1.28j |

Table 1. The optimal radial wave number \((k_r \rho_1)_{opt}|_{\text{lin}}\) for different wave numbers and flow Mach numbers.

For a given \((k_0 \rho)_{saw}\) the general formula for the optimum impedance based on Cremer’s definition can be expressed by

The general formula for the optimum impedance can formally written as equations (2)-(4),

\[
Z_{opt}|_{\text{lin}} = \theta \frac{k_r \rho_1}{\pi}
\]

In Table 2 the coefficient \(\theta\) obtained by solving equations (7) and (9) are compared, respectively.

<table>
<thead>
<tr>
<th>(k_r \rho_1)</th>
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<tr>
<td>(M_t = 0.05)</td>
<td>(M_t = 0.10)</td>
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<tr>
<td>0.95</td>
<td>0.88-0.73j</td>
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<td>1.11</td>
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<td>1.84</td>
<td>0.91-0.54j</td>
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<tr>
<td>3</td>
<td>0.94-0.47j</td>
</tr>
<tr>
<td>16</td>
<td>0.98-0.42j</td>
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Table 2. Comparison of the coefficient \(\theta\) for optimal impedance by solving Eq. (7) and Eq. (9).

Table 2 shows that the difference in optimal impedance increases when the target wave number \((k_0 \rho)_{saw}\) decreases to the cut-off regime. The coefficient \(\theta\) is also impacted by the flow Mach number. If the sound wave number \((k_0 \rho)_{saw}\) is close to the cut-off regime, the optimal impedance obtained by solving equations (14) and (15) are expected to generate higher transmission loss. In order to estimate the difference in sound attenuation, the numerical model presented by Sun et al[11] is used to compute the transmission loss with the optimal impedance given by equation (7). The computed transmission loss results are compared in Figure 3 assuming a flow Mach number of -0.1. The liner is infinitely long and uniform, as described in Figure 2(a). The line of \((k_0 \rho)_{saw} \gg 1.0\) represents the “well-cut-on” approximation, in which the optimal impedance is determined by equation (11). The \(Z_{opt}|_{\text{lin}}\) given by solving equations (14) and (15) produce much higher transmission loss compared to the results from \((k_0 \rho)_{saw} \gg 1.0\), especially in the cut-off regime.
In fact, the $Z_{\text{opt}}\mid_{\mu}$ from equations (14) and (15) is the true optimal impedance only for the given $(kr_0)_{\text{tar}}$ because the value of $(kr_0)_{\text{opt}}$ depends on the wave number $(kr_0)_{\text{tar}}$. This is the reason why the peak of the transmission loss is located at the target wave number in Figure 3. The results approach to the “well-cut-on” situation as the target wave number increases. The results in Figure 3 clearly suggest that, if there is a mean flow, the optimal impedance should be calculated by solving equation (7) in particular below or around “cut-on” range.

![Figure 3. Comparison of transmission loss from different optimal impedance for a target wave number $(kr_0)_{\text{tar}}$.](image)

In practice, liners have a finite length and reflections at the impedance discontinuity at inlet/outlet affect the TL. Figure 4 shows the comparison for one case of transmission loss including the reflections between the optimal impedance given by equations (11) and (15) assuming a flow Mach number of -0.15. The results show that the transmission loss values produced by the optimal impedance from equation (15) are much higher than the results of equation (11) in the cut-off range.

3. Optimization strategy

The basic geometry for the studied silencer is shown in Figure 1. The impedance wall provided by the proposed MPP:s silencer is composed of two parts, i.e., the MPP and the air cavity impedance. The idea to optimize the damping is now to match at one frequency this wall impedance with the Cremer impedance given by equation 15), i.e.,

$$Z_{\text{opt}}\mid_{\mu} = Z_{\text{MPP}} + Z_{\text{Cav}}.$$  \hspace{1cm} (17)

The optimal impedance can be calculated by equation (4) in the absence of flow. In order to include the influence of mean flow, equation (11) gives the optimal impedance in the “well-cut-on” regime. If the target frequency is not “well” cut-on, equation (7) should be solved to obtain the optimal radial wave number and formula (15) can be used to calculate the optimal impedance. The needed MPP impedance can be obtained from the formulas in Ref [2] and the impedance of the cavity is easily derived, see Ref [4]. The final step is then to compute the performance and here the method by Sun et al[11] is used to calculate the transmission loss of an optimized silencer.

4. Summary

A procedure to realize an optimum silencer based on the Cremer impedance is described. The procedure to find the optimal impedance of lined ducts containing mean flow in the cut-off region is presented. The Cremer impedance is created by combining a micro-perforated plate (tube) with a cavity, so that the combined impedance matches the Cremer impedance at a (given) design frequency. A high TL peak can be achieved at the design frequency due to the match of impedance to Cremer’s results. The next step in this work will be to verify the optimum design by experiments.
Allam and Åbom[3] has previously presented some measurements for non-optimum MPP:s cases, validating the modeling accuracy for the MPP silencer configuration in Figure 1.

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