Abstract

The numerical performance and accuracy of different features of a wake interaction noise prediction model have been investigated by analysing the results obtained on the geometry of an existing contra-rotating fan. The source and propagation effects have been considered separately. First, the effect of modelling the source either with a parallel or with a skew gust formulation, based on Amiet’s theory, has been investigated. The choice of the response function and its computation have been discussed. Then, the propagation formula has been modified to take into account the effect of the geometry of the blades on the retarded time of the emission.

1 Introduction

This paper describes the numerical performance and accuracy of a set of wake interaction noise prediction algorithms. These models take into account different features of the physical phenomenon, as the effect of the skewness of the incoming gust in the source formulation and the phase shift in the propagation model due to the displacement of the blade profiles. The 4.2 m diameter contra-rotating fan of the VKI low speed Eiffel-type open-jet wind tunnel L-1A, shown in figure 1, has been selected as the test case. It is driven by a variable speed DC motor of 580 kW, allowing a continuous variation of free jet velocity from 2 to 60 m/s. The free jet test section is of 3 m diameter and 4.5 m length; the contraction ratio is 4 with a typical turbulence level of 0.3 %. The wind tunnel was designed as an aeronautical wind tunnel and used for testing aircraft models, re-entry capsule models, but also for tests on racing cyclists, road vehicle models or ground structures.

2 Prediction of wake interaction noise

Since the flow ingested by a rotor is not homogeneous, each blade segment experiences a variation of both the magnitude and angle of incidence of the velocity while rotating. These variations induce a total instantaneous force \( L(t) \), which derives from the integration of the local instantaneous lift forces over the blade surface. The periodic component of \( L(t) \) is responsible for the noise emission at discrete frequencies[9]. The unsteady lift generated by the passage of the rear rotor blades in the wakes shed from the front rotor is believed to be a major source of aerodynamic noise in the present test case.
2.1 A linearized aeroacoustic theory

Figure 2 describes a two-dimensional case in which $U$ is the characteristic convection speed of disturbances along the airfoil chord line, $\overline{u}$ is the velocity fluctuation which would exist if no airfoil were present, with $u$ and $w$, the streamwise and normal components, respectively.

In the case of a slightly loaded airfoil, the normal component $w$ of the velocity fluctuations to the airfoil plane is responsible for the major part of the unsteady lift. For this reason, only this component is considered in a linearised unsteady aerodynamic theory and the airfoil is assumed to behave like a flat plate with zero incidence to the flow. Such a theory was used by Fournier [4] to compute the noise generated by helicopter ducted tail rotors. Its principle is that the tonal and broadband spectral components of noise generated by the wake interaction are distinct and hence the related aeroacoustic phenomena are themselves decoupled. For this reason, it is supposed that the tonal noise spectrum is due to the interaction of the blades with the periodic mean velocity deficit of the wakes, while the broadband noise spectrum is due to the turbulence of the wakes, as represented in figure 3. This is then a linear approach to the interaction phenomena, from both an aerodynamic and acoustic viewpoints.

2.2 Modelling of the source as a parallel gust - airfoil interaction

The simplest way to model the interaction between the blade segments and the incoming wake is to assume that the mean velocity deficit of the wake represents a parallel incompressible gust for the rear rotor. The corresponding formulation[1] for the local instantaneous lift fluctuation, $l$, is in function of the chord-wise coordinate, $x'$, and of the chord-wise wave number of the incoming gust is, that is $k_x = \frac{2\pi \lambda}{T_{pp}U}$, being $T_{pp}$ the
blade passing period related to the front rotor. For a given blade passing frequency harmonic (BPFH) the local unsteady lift is then

\[ l(k_x, x', t) = 2\pi \rho_0 U \hat{w}(k_x) g(k_x, x') \exp(-i k_x U t) \]  

where \( \rho_0 \) is the reference density, \( \hat{w}(k_x) \) is the spatial Fourier transform of the velocity fluctuation, and \( g(k_x, x') \) is the aerodynamic response function of the profile. The response function, in turn, can be expressed by using Sears’ theory for a two-dimensional flow, which is valid for compact airfoils:

\[ g_s(k_x, x') = \sqrt{\frac{1}{1 + 2x'/c} S\left(k_x c/2\right)} \]  

\[ S\left(k_x c/2\right) \]  

is the Sears’ function, which can be expressed in terms of Bessel functions, but has also a good approximation \( S_a\left(k_x c/2\right) \) as\[1\]::

\[ S_a\left(k_x c/2\right) = \frac{1}{\sqrt{1 + \pi k_x c}} \exp\left[-i k_x c/2 \left(1 - \frac{\pi^2}{2(1 + \pi k_x c)}\right)\right] \]  

which has been used to obtain the present results.

The local instantaneous lift fluctuation is then integrated along the chord and the span, \( d \), of the airfoil to get the total unsteady lift in function of the wave number:

\[ L(k_x, \omega) = \pi^2 \rho_0 c d U \hat{w}(k_x) S_a\left(k_x c/2\right) \exp\left(-i k_x U t\right) \]  

whose Fourier transform is

\[ \hat{L}(k_x, \omega) = \pi^2 \rho_0 c d U \hat{w}(k_x) S_a\left(k_x c/2\right) \delta(\omega - k_x U) \]  

\[ = \pi^2 \rho_0 c d U \hat{w}(K_x) S_a\left(K_x c/2\right) \]  

The Kronecker delta appearing in Eq.(5) means that a given frequency component of the pressure jump is produced by the \( K_x = \omega/U \) value of the chord-wise wave number[1].

### 2.3 Modelling of the source as a skew gust - airfoil interaction

The interaction between the rear rotor and the wakes shed from the front rotor is, however, more likely to be represented by a skew gust - airfoil interaction model. In fact, the incoming wake will sweep the leading edge of the rotor with a finite velocity, giving rise to the existence of a span-wise wave number. Furthermore, whereas in a turbulent gust an ideally infinite set of span-wise wave numbers, \( k_y \), is related to a single value of the chord-wise wave number, \( K_x \), in this case only one \( K_y \) value corresponds to a given \( K_x \).
Figure 4: Reduction of the three-dimensional wake - blade interaction to a series of skew gust - airfoil interaction phenomena.

In this context, the relative inclination of the rear blades with respect to the incoming wakes is taken into account by means, again, of a segmentation method which allows to reduce the three-dimensional wake - blade interaction phenomenon to a series of skew gust - airfoil interaction problems of known solution, as seen in figure 4.

Generalising the previous formulation for parallel gust, the local unsteady lift can now be expressed in the frequency domain as:

\[ \hat{\tilde{l}}(K_x, K_y, x') = 2\pi \rho_0 U \hat{\tilde{w}}(K_x, K_y) g(K_x, K_y, x') \exp(-i K_y y) \]  

(7)

where \( \hat{\tilde{w}}(K_x, K_y) \) is the double spatial Fourier transform of the velocity fluctuation. Integrating Eq.(7) over the blade segment surface yields:

\[ \hat{\tilde{L}}(K_x, K_y) = 2\pi \rho_0 U \hat{\tilde{w}}(K_x, K_y) G(K_x, K_y) \frac{i}{K_y} \left( \exp(-i K_y d) - \exp(i K_y d) \right) \]  

(8)

As for the aerodynamic response function, \( g(K_x, K_y, x') \), we have made use of Amiet’s theory for leading edge noise[1, 3], which takes into account also the compressibility and non-compactness effects that may be present in the gust - airfoil interaction problem. The response functions for subcritical and supercritical gusts are derived by first solving the scattering of noise from the leading edge of the airfoil with Schwarzschild’s technique and then adding a correction term for the back-scattering from the trailing edge, where the Kutta condition holds.

In the framework of Amiet’s theory, a gust is defined as supercritical or subcritical according to the following relation:

\[ (\bar{k}_y/\beta)^2 < (\bar{k}_x^* M)^2 \] supercritical  
\[ (\bar{k}_y/\beta)^2 > (\bar{k}_x^* M)^2 \] subcritical  

(9) (10)

where \( M = U/c_0, \beta = \sqrt{1 - M^2}, \bar{k}_x = k_x c/2, \bar{k}_x^* = \bar{k}_x/\beta^2 \) and \( \bar{k}_y = k_y c/2 \).

The aerodynamic response functions for both supercritical and subcritical regimes are given below, where the subscript 1 denotes the main leading edge term and 2 denotes the trailing edge back-scattering correction[3].

2.3.1 Supercritical gust

Defining the non dimensional chord-wise coordinate as \( \bar{x} = x/(c/2) \), the leading edge and trailing edge terms are, respectively
\[ G_{1\text{sup}}(k_x, k_y) = \frac{c}{2} \int_{-1}^{1} \frac{\exp(-i \frac{\pi}{4})}{\pi \sqrt{(k_x + \beta^2 \kappa)(\bar{x} + 1)}} \exp(-i (\kappa - M^2 \bar{k}_x^*) (\bar{x} + 1)) \, d\bar{x} \]
\[ = \frac{c}{2} \frac{(1 - i)}{\pi \sqrt{(k_x + \beta^2 \kappa)}} \theta_1 E^*(2 \theta_1) \] (11)

and

\[ G_{2\text{sup}}(k_x, k_y) \approx -\frac{c}{2} \int_{-1}^{1} \frac{\exp(-i \frac{\pi}{4})}{\pi \sqrt{2 \pi (k_x + \beta^2 \kappa)}} \left[ 1 - (1 + i) E^*(2 \kappa (1 - \bar{x})) \right] \cdot \exp(-i (\kappa - M^2 \bar{k}_x^*) (\bar{x} + 1)) \, d\bar{x} = \]
\[ = \frac{c}{2} \frac{\exp(-i \frac{\pi}{4})}{\theta_1} \frac{i (1 - \exp(-2i \theta_1)) + (1 - i) [E^*(4 \kappa) - \exp(-2i \theta_1) E^*(2 (2 \kappa - \theta_1))] }{\sqrt{2 \kappa - \theta_1}} \] (12)

where \( \kappa = \sqrt{(\bar{k}_x^*)^2 - (\bar{k}_y/\beta)^2} \), \( \theta_1 = \kappa - \bar{k}_x^* M^2 \) and \( E^*(x) = \int_0^x \frac{\exp(-it)}{\sqrt{2 \pi t}} \, dt \) is the complex conjugate of the Fresnel integral.

### 2.3.2 Subcritical gust

Defining, as for the supercritical gust, the non dimensional chord-wise coordinate as \( \bar{x} = x/(c/2) \), the leading edge and trailing edge terms are, respectively

\[ G_{1\text{sub}}(k_x, k_y) = \frac{c}{2} \int_{-1}^{1} \frac{\exp(-i \frac{\pi}{4})}{\pi \sqrt{(k_x + i \beta^2 \kappa') (\bar{x} + 1)}} \exp(i (\kappa' + M^2 \bar{k}_x^*) (\bar{x} + 1)) \, d\bar{x} \]
\[ = \frac{c}{2} \frac{(1 - i)}{\pi \sqrt{(k_x + i \beta^2 \kappa')}} \theta_2 E(2 \theta_2) \] (13)

and

\[ G_{2\text{sub}}(k_x, k_y) \approx -\frac{c}{2} \int_{-1}^{1} \frac{\exp(-i \frac{\pi}{4})}{\pi \sqrt{2 \pi (k_x + i \beta^2 \kappa')}} \left\{ 1 - \text{erf}(\sqrt{2 \kappa'(1 - \bar{x})}) \right\} \cdot \exp(i (\kappa' + M^2 \bar{k}_x^*) (\bar{x} + 1)) \, d\bar{x} = \]
\[ = \frac{c}{2} \frac{\exp(-i \frac{\pi}{4})}{\theta_2} \frac{\exp(2i \theta_2) \text{erf}(\sqrt{2 (2 \kappa' + i \theta_2)}) - \text{erf}(\sqrt{4 \kappa'}) - (\exp(2i \theta_2) - 1)}{\sqrt{2 \kappa' + i \theta_2}} \] (14)

where \( \kappa' = \sqrt{\bar{k}_y/\beta^2 - (\bar{k}_x^*)^2} \), \( \theta_2 = \kappa' + \bar{k}_x^* M^2 \), \( E(x) = \int_0^x \frac{\exp(it)}{\sqrt{2 \pi t}} \, dt \) is the Fresnel integral and \( \text{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} \frac{\exp(-t)}{\sqrt{t^4}} \, dt \) is the error function.

From a physical point of view, a smooth transition of the response function from the supercritical to the subcritical regime is expected. However, a discontinuity is observed at \( \kappa = 0 \), which is due to the fact that an exact analytical solution could be reached only by infinite iterations of Schwarzschild’s method[10]. For this reason, a numerical regularization technique has been adopted, matching the values of the response functions at \( \kappa = 0 \). A comparison between analytical and regularized response functions for an airfoil of chord \( 2b = 0.3 \, m \) and an incoming gust of advection speed \( U = 77 \, ms^{-1} \) and frequency of 167 Hz is shown in figures 5 and 6.
Figure 5: Analytical and regularized aerodynamic response function - magnitude.

Figure 6: Analytical and regularized aerodynamic response function - phase.
The parallel gust-airfoil interaction can be considered a special case of the skew gust formulation and be solved by simply using in Eq. (6) Amiet’s response functions with \(K_y = 0\) instead of Sears’ function:

\[
\hat{L}(k_x, \omega) = \pi^2 \rho_0 c d U \hat{w}(K_x) G_{sup}(K_x, K_y = 0).
\] (15)

The effect of using a different response function for the parallel gust formulation will be discussed in section 4.1.

### 2.4 Noise propagation formulation

The far-field noise has been computed by using the formulation of Ffowcs Williams and Hawkings (FWH) analogy [5] with Bessel functions. The acoustic pressure, \(p'\), is computed at locations equally spaced on the surface of a sphere as

\[
p'n_B \sim -i B k_{nB} e^{-ik_{nB}x} \frac{x}{4\pi} \sum_{p=-\infty}^{+\infty} e^{-i(nB-p)(\phi-\phi_0-\pi/2)} \left( J_{-nB+p}(-k_{nB} R_0 \sin \theta) F_p^{(T)} \cos \theta - \right.
\]

\[
- \frac{nB-p}{k_{nB} R_0} \left. J_{-nB+p}(-k_{nB} R_0 \sin \theta) F_p^{(D)} - -i J_{-nB+p}(-k_{nB} R_0 \sin \theta) \sin \theta F_p^{(R)} \right),
\] (16)

where \(p'_n\) is the sound pressure perturbation emitted at a given BPFH of the rear rotor; \(B\) is the number of rear blades, so this formula represents a constructive interference for which the total sound emitted by the fan is \(B\) times the sound of a single blade; \(x\) is the listener’s position radius; \(p\) represents the blade loading harmonics (BLH), so the noise emitted at a given BPFH is the sum over all the BLHs; \(\phi\) and \(\theta\) are the listener’s azimuthal and polar angle, respectively; \(R_0\) is the radius where the force is applied; \(J_{-nB+p}\) is a Bessel function of the first kind which modulates the Doppler frequency shift during blade revolution; finally, \(F_p^{(T)}, F_p^{(R)}, F_p^{(D)}\) are the tangential, radial and axial components of the unsteady lift, respectively.

The sound pressure perturbation is thus computed for each dipole at every point on the spherical surface and for all the BPFHs up to the ninth. Then the contributions of the different sources are summed and the sound power is integrated over the sphere.

In the formulation developed so far, we have assumed that the center of force of the unsteady lift of each blade strip lies on a radial line. In reality, however, the centres of force are relatively displaced both in the azimuthal and axial direction. The importance of this effect will be assessed in section 4. The relative azimuthal displacement of the blade strips can be accounted for in the far-field formula with the term \(\phi_0\), which represents a phase shift in the frequency domain, corresponding to a retarded time of emission, whereas the radial coordinate of the dipole location is subtracted to that of each point of the sphere.

### 3 Generation of synthetic wake data

The shape of the wakes shed from the front rotor can be approximated by using a Gaussian function, which is defined by the mean velocity deficit in the wake centerline, \(\mu_c\), and by the wake width, \(Y_S\), as outlined in figure 7. As a consequence, the periodic velocity disturbance that sweeps the rear rotor can be modelled by:

\[
w\left( \frac{x'}{U_0}, t \right) = \overline{w} \sum_n f_s \left( \frac{x'}{U_0} - t + nTpp \right)
\] (17)

where \(\overline{w}\) is the upwash component of the mean velocity deficit, \(\mu_c\), and \(n\) is the number of blades of the front rotor. The periodicity is achieved thanks to the choice of a function \(f_s\) slightly different from 0 outside the wakes, which is

\[
f_s(t) = \exp \left[ -\frac{\chi t^2}{\tau_s^2} \right]
\] (18)
where $2\tau_s = 2Y_S/\Omega R$. The parameter $\chi$ has been set equal to 12 by Fournier [4] in agreement with the experimental results of Raj and Lakshminarayana [8], who investigated the development of three-dimensional turbulent wakes behind rotors of axial turbomachinery.

In order to calculate the unsteady lift, the function describing the mean velocity deficit of the wake (Eq. (17)) is decomposed in Fourier series as

$$w(x',t) = \bar{w} \sum_{\lambda} F(\lambda) \exp \left[ i \left( \frac{2\pi \lambda}{T_{pp}} \left( \frac{x'}{U_0} - t \right) \right) \right],$$

where $2\pi T_{pp}$ is the blade passing angular frequency. The Fourier coefficients, $F$, are

$$F(\lambda) = \frac{\tau_s}{T_{pp}} \sqrt{\frac{\pi}{\chi}} \exp \left[ -i \frac{\pi^2 \lambda^2 \tau_s^2}{\chi T_{pp}^2} \right].$$

In a previous work [6], the parameters $\mu_c$ and $Y_S$ were extracted from CFD data. Steady RANS computations have been performed with a mixing plane between the two rotors, where the flow quantities are pitchwise averaged, and for this reason the parameters had to be extrapolated to provide their value at the leading edge of the rear rotor, based on the empirical correlations of Raj and Lakshminarayana [8]. According to these Authors, the decaying of the defect in mean velocity at the wake centerline is given by

$$\mu_c U_s = \exp \left[ -\frac{\pi^2}{14} \left( \frac{x}{S} + 3.46 \right) \right] \text{ for } x/S > 0.15$$

where $U_s$ is the mean velocity in the streamwise direction and $S$ is the pitch of the blades. The trend of the wake width variation is in its turn

$$\frac{Y_S}{Y_{S0}} = 1.61 \left( \frac{x}{S} \right)^{0.23}$$

where $Y_{S0}$ is the wake width taken at the first extraction plane.

Unfortunately, this extrapolation procedure does not yield any information about the three-dimensional shape of the wake, which is necessary for the application of the skew gust formulation. For this reason, instead of extracting the wake characteristics from CFD data, synthetic wake data have been generated for the noise calculations presented in this work. In this case, the shape of the wake is given by Eq.s (17) and (18), whereas the variation of the mean velocity deficit and of the wake width is controlled by Eq.s (21) and (22), respectively. Figure 8 depicts the synthetic wake, as seen in the secondary plane, generated at two different axial locations. As a result, the wake parameters vary continuously along the radial direction and the three-dimensional shape of the wake is available for the acoustic post-processing. Anyway, there are more complex physical phenomena which are not taken into account in this empirical model, as already remarked by other Authors [2].

4 Analysis of the results

4.1 Comparison of parallel and skew gust formulations

In this section we have made use of the three source models: parallel gust with Sears’ (Eq. (6)) or Amiet’s response function (Eq. (15)) and skew gust (Eq. (8)), in order to compute the distribution of unsteady lift along the radius of the rear rotor blades of the L-1 ventilator, visible in figures 9 and 10. In figure 9, the unsteady lift is divided by the area of the corresponding blade strip, which allows to perform convergence tests on the number of strips (see section 4.3). The unsteady lift density is then normalized by the product of the reference density of air, $\rho_0 = 1.184$ kg m$^{-3}$, and the square of the mean flow velocity, $U_0 = 26.2$ ms$^{-1}$. All the curves are related to the base blade loading frequency.
Figure 7: Geometric representation of the wake [4].

Figure 8: Contours of the mean velocity deficit of the wake in the secondary plane at different axial locations.

Within the framework of a parallel gust approach, the difference between the results obtained with Sears’ and Amiet’s response functions can be explained with the presence of non-compactness effects, since the minimum value of $k_z c$, found at the blade tip for the first BPF, is already equal to 2.8 (based on a mean chord $c = 0.3$ m, whereas Sears’ theory requires compactness in a much stricter sense[1]. The skew gust formulation, in its turn, yields a significantly lower unsteady lift magnitude. It can be also observed that the incoming gust is seen as subcritical near the hub, whereas it is supercritical on most of the blade extension going up to the tip. This is a kind of essential information on the wake - blade interaction problem which is lost by using a parallel gust theory. The distribution of the phase of the dipoles along the radius of the rear rotor, depicted in figure 10, shows a much larger interval for Sears’ theory than for Amiet’s, which means also enhanced cancellation effects.

Figure 11 depicts the tonal noise spectra computed with the three source models, having applied the phase shift correction in the propagation formula of Eq. (16). As expected, the skew gust model yields the lowest sound power level (SWL) at all frequencies. Within the parallel gust approach, a lower SWL is obtained with the Sears’ response function, although the magnitude of the unsteady lift is bigger. This is due to the aforementioned cancellation effects related to the distribution of the phase of the dipoles along the radius.
Figure 9: Comparison of the magnitude of the normalized unsteady lift density, computed with the three source models.

Figure 10: Comparison of the phase of the unsteady lift density, computed with the three source models.

4.2 Effect of the phase shift in the propagation formulation

The following plots illustrate the effect of the azimuthal and axial displacement of the centres of force in noise propagation model. The tonal noise spectra have been computed for a given source model with and without the correction for the related retarded time of the emission.

As shown in Figure 12, for the parallel gust - Sears’ response function model, the phase shift correction results in a redistribution of the acoustic energy among the different frequencies. For both parallel and skew gust with Amiet’s response function, depicted in figures 13 and 14, respectively, the phase shift correction causes a decrease of the SWL especially above the sixth harmonic.

4.3 Convergence of the noise prediction with the number of blade strips

The advantage of using a synthetic wake model is that all the parameters, that contribute to the computation of the unsteady lift density, vary continuously along the blade span. For this reason, a consistent description of the source can be obtained for different numbers of blade strips, as can be seen in figure 15 for the skew
Figure 11: Comparison of far field spectra computed with the three source model; the propagation formulation has been corrected for the phase shift.

Figure 12: Effect of the phase shift correction on the computation of the far field spectrum using a parallel gust - Sears’ response function source model.

gust model. As for the corresponding SWL spectrum, depicted in figure 16, thirty blade segments have been sufficient to reach a convergent prediction. It can be noticed that the difference between the predictions achieved with different numbers of strips is very small at the lower frequencies and increases at the higher frequencies. A similar behaviour of the convergence has been observed for the other source models.
Figure 13: Effect of the phase shift correction on the computation of the far field spectrum using a parallel gust - Amiet’s response function source model.

Figure 14: Effect of the phase shift correction on the computation of the far field spectrum using a skew gust source model.
Figure 15: Magnitude of normalized unsteady lift density computed for different numbers of strips according to the skew gust model.

Figure 16: Far field noise spectrum computed for different numbers of strips according to the skew gust model. The propagation formula (Eq. (16)) has been corrected for the phase shift.
5 Conclusions and future work

In this work, a set of algorithms of increasing physical accuracy for the prediction of wake interaction tonal noise has been tested on the geometry of an existing contra-rotating fan. From the results related in the previous sections, it appears that a parallel gust model of the source of noise can not be considered as a reliable surrogate of a more complex skew gust model and that a careful choice of the response function must be made in order to take into account compressibility and non-compactness effects. Furthermore, not taking into account the retarded time effects in the propagation model can lead to a over-estimation of the far field noise, especially at the higher frequencies. Finally, the synthetic wake generation algorithm, that has provided the data for the noise computations, seems the most promising way to apply the wake interaction noise theory to a multi-disciplinary shape optimization of the contra-rotating fans [6, 7] and for this purpose it will be validated by comparison with the results of a time-dependent simulation.

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References


