Acoustical behavior of purely reacting liners

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This paper investigates the acoustical behavior of a purely reacting liner in presence of a grazing flow. This device exhibits an unusual acoustical behavior: for a certain range of frequencies, no wave can propagate against the flow. The effect of shear flow is investigated by the Chebyshev Spectral Method, which provides detailed information near the wall. A negative group velocity is found in a range of frequencies and it is demonstrated that the sound can be stopped.

I. Introduction

Liners with a very small resistance had already shown particular behavior with flow: It has been demonstrated that this kind of liner can be unstable\(^1\),\(^2\). In this paper, we investigate both theoretically and experimentally that these liners exhibit another unusual acoustical behavior: For a certain range of frequencies and Mach numbers, no wave can propagate against the flow.

II. Propagation in the reacting lined ducts without flow

First, we consider the modes in 2D lined duct without flow where the upper wall is rigid and the lower wall is lined. The acoustic pressure is written:

\[ p(x, y) = p(y)e^{(j\omega t - jkx)}, \]  

(1)

and \( p(y) \) verifies the Helmholtz equation:

\[ \frac{d^2p(y)}{dy^2} - \alpha^2 p(y) = 0, \]  

(2)

where \( \alpha^2 = k^2 - \omega^2 \). Note that the equations are written in dimensionless form where \( p \) is normalized by \( \rho_0 c_0^2 \), \( x \) and \( y \) by the height of the duct \( H \), \( t \) by \( H/c_0 \), and then \( \omega = 2\pi f H/c_0 \) is the Helmholtz number.

At the rigid wall \( (y = 1) \), the normal acoustic velocity \( v \) vanishes. At the lined wall \( (y = 0) \), the boundary condition is given by \( v = Yp \). For a purely reacting liner made with ideal tubes, \( Y \) is equal to \( Y(\omega) = -j \tan(\omega b) \), where \( b \) is the reduced thickness of the liner.

The above equations lead to the dispersion relation:

\[ \alpha \tanh(\alpha) = \omega \tan(\omega b). \]  

(3)

In the following, we only consider sufficiently low frequencies, for which only plane wave can propagate in the rigid duct \( (\omega < \pi) \). In the very low frequencies \( (\omega \ll 1 \text{ and } \alpha \ll 1) \), we have \( \alpha^2 = b \omega^2 \) and \( k = \pm (1 + b)^{1/2} \omega \). The reduced phase velocity \( c_\phi = \omega/k \) is then smaller than 1 (the phase velocity is smaller than the sound velocity). In Fig. 1, \( c_\phi \) is plotted as a function of \( \omega \). It can be seen from this figure that the phase velocity \( c_\phi \) is always smaller than 1 and tends to zero when \( \omega \) approaches \( \omega_R = \pi/(2b) \) corresponding to the quarter wavelength resonance of the liner.
Figure 1. Phase velocity \( c_\phi \) as a function of the Helmholtz number \( \omega \).

The wave becomes more and more concentrated toward the wall and the wavelength becomes shorter and shorter when the frequency approaches \( \omega_R \). This is illustrated in Fig. 2 where the pressure field is computed for \( \omega = 0.412, \ b = 10/3 \) and for a reduced length of the liner \( L = 40/3 \).

Figure 2. Pressure field for \( \omega = 0.412, \ b = 10/3 \) and \( L = 40/3 \).

Thus in a lined duct with a purely reacting impedance, the sound propagates slower than in a rigid duct. It can be imagined that, if the sound is slow enough, a counter flow can stop the sound even if the Mach number is smaller than 1. That is what we have investigated in the next sections.

III. Propagation in the reacting lined ducts with a uniform flow

The first step to investigate the flow effect is to consider a uniform flow. In this case the propagation equation is simple but there can be problems in the boundary condition. Here, we use the classical Ingard-Myers condition\(^3\) even if there are some doubts on its validity and alternative boundary conditions had been recently derived\(^4\)–\(^7\)

With uniform flow, the pressure verifies again the Helmholtz equation:

\[
\frac{d^2 p(y)}{dy^2} - \alpha^2 p(y) = 0, \tag{4}
\]

but the transverse wave number \( \alpha \) takes into account the convection by \( \alpha^2 = k^2 - (\omega - kM)^2 \) where \( M \) is the Mach number. At the rigid wall \( (y = 1) \), the normal acoustic velocity \( v \) still vanishes. But, at the lined wall \( (y = 0) \), the boundary condition, taking into account the continuity of the pressure and of the normal displacement, is given by

\[
v = \frac{\omega - kM}{\omega} Y_p. \tag{5}
\]

The dispersion relation becomes:

\[
\alpha \tanh(\alpha) = (\omega - kM)^2 \frac{\tan(\omega b)}{\omega}. \tag{6}
\]
The two sides of Eq. (6) are plotted in Fig. 3 for \( \omega = 0.1, b = 10/3 \) and \( M = 0.2 \). The Eq. (6) has now 4 propagative solutions \((k \in \mathbb{R})\). These 4 solutions can be split into two categories: 2 solutions are close to zero (in the zoom of Fig. 3) and 2 solutions have much larger values of \( k \).

Figure 3. The two sides of Eq. (6) for \( \omega = 0.1, b = 10/3 \) and \( M = 0.2 \). Left hand side in blue, right hand side in green.

For the two solutions close to 0 when \( \omega \) is small, we can consider that \( \alpha \tanh(\alpha) \approx \alpha^2 \). Then these solutions are approximated by

\[
\tilde{k}_1 = \frac{\sqrt{1 + \beta \omega}}{1 + \sqrt{1 + \beta M}} \quad \tilde{k}_2 = -\frac{\sqrt{1 + \beta \omega}}{1 - \sqrt{1 + \beta M}} \quad \text{where} \quad \beta = \frac{\tan(\omega b)}{\omega} \quad \text{as} \quad \omega \to 0
\]

Those solutions are close to the solutions without flow. For the other two solutions, we can suppose that \( \alpha \tanh(\alpha) \approx \alpha \approx |k|\sqrt{1 - M^2} \) and the solutions are at the intersections of the straight lines representing the left hand side of Eq. (6) (green lines in Fig. 3) and of the parabola representing the right hand side of Eq. (6) (blue line in Fig. 3).

\[
\tilde{k}_3 = \sqrt{1 - M^2} + 2M\omega\beta + \sqrt{1 - M^2 - 4M\omega\beta \sqrt{1 - M^2}} \quad \text{as} \quad \omega \to 0 \frac{1 - M^2}{bM^2}
\]

\[
\tilde{k}_4 = -\sqrt{1 - M^2} + 2M\omega\beta - \sqrt{1 - M^2 - 4M\omega\beta \sqrt{1 - M^2}} \quad \text{as} \quad \omega \to 0 \frac{1 - M^2}{bM^2}
\]

Those two solutions exist only when flow is present. They are very dependent on the boundary condition used. For example, if the continuity of normal velocity is used instead of the continuity of the normal displacement, Eq. (5) becomes \( v = Y_p \). In this case, the right hand side of Eq. (6) becomes a straight line instead of a parabola and Eq. (6) have only two solutions just like in the case without flow.

The variation of the 4 solutions as a function of \( \omega \) is given in Fig. 4 and compared to the approximated solutions given by Eq. (7) and Eq. (8). It can be seen that the two positive solutions \( k_3 \) and \( k_4 \) merge at the resonance frequency of the liner (\( \omega = \omega_R \)). When they merge, the wave number is equal to \( \omega/M \). The two negative solutions, propagating against the flow, merge at a frequency \( \omega_M \) which is lower than \( \omega_R \).

This confirms the hypothesis put forward in the previous section: Above a reacting liner, a sound wave can be stopped by a counter flow even if the Mach number is smaller than 1.
Figure 4. Variation of the propagative wave numbers $k$ as a function of the Helmholtz number $\omega$ for $b = 10/3$ and $M = 0.2$. The continuous lines represent the solutions of Eq. (6) (dark blue = $k_1$, dark green = $k_2$, light blue = $k_3$, light green = $k_4$). The orange dashed lines represent the 4 approximated solutions given by Eq. (7) and Eq. (8). The black dashed line is $\omega - kM = 0$.

The variation of the stopping frequency $\omega_M$ as a function of the Mach number is given in Fig.5. The range of frequencies where no counter propagative wave exists is linearly increasing from $M = 0$ to $M \approx 0.45$. For $M > 0.45$, no counter propagative wave exists practically in all the frequency range.

Figure 5. Variation of the stopping frequency $\omega_M$ as a function of the Mach number for $b = 10/3$.

It can be seen from the Fig. 4 that the group velocity $c_g = d\omega/dk$ for modes 3 and 4 has a sign opposite to the phase velocity sign ($c_\varphi = \omega/k$). Thus those modes can be considered as negative energy waves. When the modes merge (i.e. when $\omega = \omega_R$ for $k > 0$ and $\omega = \omega_M$ for $k < 0$), the group velocity is equal to 0.

It has been pointed out that the behavior of the stopping point and of the mode is very dependent on the boundary condition, we will investigate in the next section the effects of a shear flow on the behavior of a reacting liner.

IV. Propagation in the reacting lined ducts with a shear flow

With shear flow, the equation governing the propagation of wave is given by the Pridmore-Brown equation:

$$ (\omega - kM) \left( \frac{d^2p}{dy^2} + (\omega - kM)^2 p - k^2p \right) + 2k \frac{dM}{dy} \frac{dp}{dy} = 0 $$

The associated boundary conditions are $v = 0$ at the rigid wall ($y = 1$) and $v = Yp$ at the lined wall ($y = 0$) if we suppose that the mean flow is equal to 0 at the lined wall.
To study the influence of the boundary layer thickness, we choose a piece-linear profile. The Mach number varies linearly from 0 to $M_0$ when $y$ varies from 0 to $\varepsilon$ and is constant and equal to $M_0$ for $\varepsilon < y < 1$ (see Fig. 6).

![Figure 6. Description of the shear flow profile.](image)

The Chebyshev spectral method is used to solve the problem. The discretization in the $y$-coordinate is chosen such that there are at least 12 points in the boundary layer. The results for 3 values of $\varepsilon$ are shown in Fig. 7.

![Figure 7. Wave number as a function of $\omega$ for 3 different values of the boundary layer thickness for $b = 10/3$ and $M_0 = 0.2$. Blue: $\varepsilon = 0.1$, red: $\varepsilon = 0.01$ and green: $\varepsilon = 0.001$. The black lines are the solutions for uniform flow.](image)

A big difference between the uniform and shear flow cases is the presence of a continuum of hydrodynamic modes. $k = \omega/M$ is a singular solution of Eq. (9) for all $M$ between 0 and $M_0$. Thus all $k > \omega/M_0$ are solutions of the dispersion relation (gray area in Fig. 7). The Mode 1 for which $0 < k < \omega/M_0$ is practically unchanged by the presence and the shape of the boundary layer.

The waves propagating against the flow are very sensitive to the shape of the boundary layer. The number of propagative modes is changed by the effect of the boundary layer. This effect has been already noted by Brambley. Considering the case $\varepsilon = 0.01$ in Fig. 7, we can see that there is only one wave that can propagate against the flow for $0 < \omega < \omega_1$. This wave is close to the mode 2 in Fig. 4. For $\omega_1 < \omega < \omega_2$, three waves exist. Two of them are close to the mode 2 and 4 in the uniform case and another wave appears with large value of $k$. For $\omega_2 < \omega < \omega_R$, only the new wave with very large values of $k$ and small values of $c_g$ remains.

When the boundary layer thickness increases, the wave for which the group and the phase velocities are opposite in sign will disappear (see the case $\varepsilon = 0.1$ in Fig. 7). In this case, the points with a zero group velocity also disappear. On contrary, when the boundary layer thickness decreases, two of solutions converge toward the uniform case and the new solution has larger and larger values.
V. Comparison with experimental results

The material was designed to act locally and to have a resistive part as small as possible. The tested material was an honeycomb (cell size = 4 mm, height = 50 mm) covered by a very pervious fine mesh tissue (flow resistance $\sim 0.02\rho_c$). This material is mounted flush to the flow. The sample has a size 100 $\times$ 200 $\times$ 50 mm.

This sample is insert in the flow test bench of LAUM which has been described in previous works. The measurement section is rectangular (height = 15 mm, width = 100 mm). The flow in this section has a maximum mean value of 0.3. Two main measurement types are done: 1) Using the upstream and downstream microphones (6a and 6b in Fig. 8), the transmission and reflection coefficients with and against the flow can be determined by a 2 sources method, 2) Using the 11 equally spaced (distance 20 mm) microphones located on the wall opposite to the sample, the less attenuated wave numbers in the lined section are determined for a wave propagating with the flow and against the flow.

![Figure 8. Schematic description of the experimental setup. 1: compressor, 2: flowmeter, 3: anechoic terminations, 4: acoustic sources, 5: temperature sensors, 6: microphones.](image)

A. Results for transmission and reflection coefficients

![Figure 9. Experimental results of the transmission (a) and reflection (b) coefficients for $b = 10/3$ and $M_0 = 0$ and 0.2. Red: without flow, blue: M=0.2 incident wave in the flow direction, cyan: M=0.2 incident wave against the flow.](image)

The experimental results of the transmission and reflection coefficients are depicted in Fig 9. It can be seen that the transmission without flow decreases for frequency far lower than the predicted frequency $\omega_R$. This effect is due to dissipation that exists in the material even if we try to reduce it as far as possible. The residual dissipation will be the main cause of discrepancies between the above theory and the experimental results.

We can see, for the results with flow, that the transmission is stopped against the flow for a frequency higher than the predicted frequency $\omega_M$ but lower than the frequency where the wave propagating with flow is stopped. It means that there is a range of frequency where the waves can propagate in the flow direction but not in the the other direction. In this frequency range, an "acoustical diode" is created.


B. Result for the wave numbers

![Figure 10](image)

**Figure 10.** Real part of the pressure measured by the 11 microphones in front of the material normalized by the incident pressure. Positions 0 and 10 correspond to each extremity of the material (spacing between microphones is 20 mm). The green arrow indicates the incident wave. The blue arrow indicates the flow direction. (a) and (b): $M = 0$, (c): $M = 0.2$ incident wave in the flow direction, (d): $M = 0.2$ incident wave against the flow. Note that the pressure is only known at the 11 microphones position, the values between the lines come from an interpolation.

From the pressure measured at the 11 microphones located in front of the material (see Fig 10), we can extract the wave number of the least attenuated modes. The real part of the wave number is shown in Fig. 11.

![Figure 11](image)

**Figure 11.** Variation of the measured wave numbers as a function of the Helmholtz number $\omega$ for $b = 10/3$ and $M=0$ and 0.2. Red symbols: without flow, blue symbols: $M=0.2$ incident wave in the flow direction, cyan symbols: $M=0.2$ incident wave against the flow. The magenta lines are the prediction without flow and the green lines are the prediction for $M = 0.2$. The experimental curves are stooped when the transmission is smaller than 0.05.

We can see that the experimental values of $k$ are in reasonable agreement with the predictions except for the wave against the flow. Again, most of this discrepancy can be attributed to the dissipation which is bigger when the wave propagates against the flow as we can see from the imaginary part of $k$ given in Fig 12. Another part can be attributed to the boundary condition with flow and to the boundary layer effects.
VI. Conclusion

Purely reacting liners produce waves that are concentrated along the liner and propagate slower than the sound velocity. When a flow is added the waves that propagate against the flow can be cut off. This produces a frequency band acting like an acoustical diode. This effect is very sensitive to the flow boundary layer and to the remaining dissipation in the liner.

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References