APPLICATION OF A HIGH-ORDER FEM SOLVER TO REALISTIC AEROENGINE EXHAUST NOISE RADIATION

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For many industries the propagation of sound in complex flows is a critical issue. Most Computational Aero Acoustics propagation methods currently in use in industry are based on the full potential theory which cannot properly describe sound propagation through complex sheared flows. When dealing with turbomachinery noise radiated from the engine exhaust, a strong refraction occurs through the jet shear layer. To better represent the physics at hand one can solve instead the Linearised Euler Equations (LEE). However, time-domain LEE models have shortcomings for industrial applications, such as the presence of linear instabilities and the stability of impedance models. Most of these issues can be avoided by solving the LEE in the frequency domain. Nevertheless, this can be very demanding because of the high memory requirements associated with solving large sparse linear systems. For high frequency, the standard finite element method (FEM) is known to suffer from large dispersion errors. Its straightforward application to the LEE, which involve up to five unknowns in 3D, would be computationally costly. To address these issues, an alternative approach based on high-order FEM is presented in this paper. A discretised axisymmetric form of the LEE is described in conjunction with Perfectly Matched Layers. In addition, a numerical stabilisation scheme of type Galerkin/Least-Squares is included in the numerical model. The proposed method is applied to the propagation of duct modes inside a turbofan engine exhaust, with complex geometry and non-uniform mean flow. The sound propagation and radiation are accurately described, as well as the interaction between the acoustic waves and the hydrodynamic field resulting in the vorticity shedding from the duct lips.

1. Introduction

The prediction of acoustic propagation and radiation is an important concern in many industrial applications involving air flows. In particular a critical application of aerospace engineering lies in the prediction of the sound generated by turbofan engines [1]. An overview of the main issues related to acoustic propagation on subsonic mean flows, with application to turbofan engines, is presented in [2]. Different models are available for solving the acoustic propagation in the presence of mean flows: for instance the full potential formulation for irrotational flows [3], the Pridmore-Brown equation for non-uniform unidirectional flows [4], and the convected Helmholtz wave equation for uniform mean flows [5]. Each of these formulations have advantages and drawbacks, but are not as general as the
Linearised Euler Equations (LEE). The effects of complex multidirectional sheared flows and the strong refraction through shear layers are described with the LEE. This is crucial when dealing with turbomachinery noise radiated through the engine exhaust. Another advantage of the LEE is that they support the acoustic, entropy and hydrodynamic waves, and their interactions.

The LEE can be solved either in the time domain or in the frequency domain. An overview of the challenges for time and frequency domain LEE solvers is discussed by Angeloski et al. [6]. Time domain solvers have shortcomings for industrial applications such as the complexity in modeling impedance boundary conditions [7]. They are also known to suffer from linear instabilities. In fact, the LEE do not take into account the non-linear and viscous terms that normally attenuate the Kelvin-Helmholtz instabilities in real flows. These growing instabilities may be triggered in time domain solvers, which would deteriorate the numerical solution. An option to circumvent these instabilities is to solve the LEE in the frequency domain, as suggested by Agarwal in [8].

A common choice for solving the LEE on unstructured grids is the Finite Element Method (FEM). But, the use of a direct solver in the frequency domain can lead to serious constraints in terms of computing time and memory. As an alternative, we propose to solve the LEE using a high-order FEM (p-FEM) [9]. Whereas the standard FEM is based on linear or quadratic shape functions, the p-FEM uses high-order polynomials. A well-known benefit of high-order approximations is to have larger elements and to reduce the number of degrees of freedom of the problem [10]. The FEM is known to suffer from stability issues in convection-dominated problems. The conventional Galerkin formulation experiences a lack of diffusion [12]. This can be corrected by adding some artificial diffusion terms in the formulation of the problem, such as in the Galerkin/Least-Squares (GLS) and in the Streamline-Upwind Petrov-Galerkin (SUPG) methods [13]. The stabilisation schemes developed for steady advective diffusive problems have already been used for solving the LEE in the frequency domain using linear FEM [14]. They have been shown to yield good results.

One crucial aspect when designing a computational scheme for exterior propagation is the choice of non-reflecting boundary conditions (NRBC). The governing equations are solved within a finite domain, and appropriate boundary conditions must be imposed on its borders, so that the outgoing waves are not reflected back inside the domain. Different types of NRBC have been developed. Amongst them, the Perfectly Matched Layer (PML) technique, introduced by Bérenger [11] for the absorption of electromagnetic waves, is widely used.

In this paper, a stabilised discretised axisymmetric form of the LEE is developed in conjunction with PML equations at the inlet and at the far-field boundaries. The solver is applied to a realistic engine exhaust test case: an acoustic wave is propagating through the exhaust nozzle and radiating outside. Two configurations are studied: with no-flow and with non-uniform flow. The results in the near-field are compared to the results obtained by Iob et al. [15]. This paper is organized as follow: the governing equations and the numerical model are explained in Sections 2 and 3. The results are then presented in Section 4.

2. Linearised Euler Equations

The equations of motion are written in the cylindrical space coordinates \((r, \theta, z)\) for a perfect gas, with isentropic disturbances, no viscous effect (inviscid fluid), no heat transfer and no external source. The variables are the mass density \(\rho\), the momentum vector \(\rho u\) and the non-dimensional pressure defined by Goldstein [16], \(\pi = (p/p_\infty)^{1/\gamma}\), with \(p\) the pressure, \(p_\infty\) a reference pressure and \(\gamma\) the ratio of the specific heats. After linearisation, the LEE are written for the time-harmonic \((e^{+jwt})\) dependence) perturbations \(q' = \{\rho'; (\rho u_r)' ; (\rho u_\theta)' ; (\rho u_z)' ; \pi'\}\). The square matrices \(A_r\), \(A_\theta\), \(A_z\) and \(A_c\) contain the coefficients of the equations. The LEE read:

\[
 j\omega q' + \frac{1}{r} \frac{\partial (r A_r q')}{\partial r} + \frac{1}{r} \frac{\partial (A_\theta q')}{\partial \theta} + \frac{\partial (A_z q')}{\partial z} + \frac{1}{r} A_c q' = 0 ,
\]
with,

\[
A_r = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
-u_0^2 & 2u_0 & 0 & 0 & \rho \omega^2_\tau \pi_0 \\
-u_0u_\theta & u_\theta & u_0 & 0 & 0 \\
-u_0u_\tau & u_\tau & 0 & u_0 & 0 \\
-\frac{\rho_0}{\rho_0}u_0 & \frac{\rho_0}{\rho_0} & 0 & 0 & u_0 \\
\end{bmatrix}, \quad A_\theta = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 \\
-u_0u_\theta & u_\theta & u_0 & 0 & 0 \\
-u_0^2 & 0 & 2u_\theta & 0 & \rho \omega^2_\tau \pi_0 \\
-u_0u_\tau & 0 & u_\tau & u_\theta & 0 \\
-\frac{\rho_0}{\rho_0}u_0 & 0 & \frac{\rho_0}{\rho_0} & 0 & u_\theta \\
\end{bmatrix}, \\
A_z = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
-u_0u_\theta & u_\theta & 0 & 0 & 0 \\
-u_0u_\tau & 0 & u_\tau & 0 & 0 \\
-u_0^2 & 0 & 0 & 2u_\theta & \rho \omega^2_\tau \pi_0 \\
-\frac{\rho_0}{\rho_0}u_0 & 0 & 0 & \frac{\rho_0}{\rho_0} & u_\theta \\
\end{bmatrix}, \quad A_c = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \frac{\rho \omega^2_\tau \pi_0}{\pi_0} & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}.
\]

For axisymmetric mean flows and geometries, a Fourier decomposition of the fields yields azimuthal contributions of the form:

\[
q'(r, \theta, z, t) = q'(r, z) e^{-jm\theta} e^{j\omega t},
\]

where \(m\) is the azimuthal mode. The differential operator for the LEE, written for the variable \(q'(r, z)\), reads:

\[
\mathcal{L}(q') = j\omega q' - \frac{m}{r} A_\theta q' + \frac{1}{r} A_c q' + \frac{1}{r} \frac{\partial (r A_r q')}{\partial r} + \frac{\partial (A_z q')}{\partial z} = 0.
\]

### 3. High-order Finite Element Method

The weak form of the weighted residual formulation is derived from Eq. (5). The numerical stabilisation is accounted for through an additional term. The domain \(\Omega\) is partitioned into triangular non-overlapping finite elements \(\Omega_i\). The formulation reads:

\[
\iint_\Omega \left( j\omega w^T q' - \frac{m}{r} w^T A_\theta q' - \frac{\partial w^T}{\partial r} A_r q' - \frac{\partial w^T}{\partial z} A_z q' + w^T A_c q' \right) \, dr \, dz + \sum_i \iint_{\Omega_i} \mathcal{D}(w)^T \tau_i \mathcal{L}(q') \, d\Omega_i = - \int_\Gamma rw^T (n_r A_r + n_z A_z) q' \, d\Gamma, \quad \forall w \in \mathcal{W},
\]

with \(w\) the weighting functions defined in the test function vector space \(\mathcal{W} \subset H^1(\Omega)\), and \(n_r\) and \(n_z\) respectively the \(r\)-component and the \(z\)-component of the unit normal vector \(n\) to the boundary line \(\Gamma\). The superscript \(^T\) denotes the Hermitian transpose. The matrix \(F = n_r A_r + n_z A_z\) represents the fluxes of \(q'\) along the \(n\)-direction. In the left-hand side of the formulation the stabilisation term is evaluated within each element and depends on two main features: the stabilisation scheme that is defined by the stabilisation operator \(\mathcal{D}(w)\) and the stabilisation parameter \(\tau_i\). In this paper the Galerkin/Least-Squares (GLS) stabilisation operator is used. For the axisymmetric LEE, this operator reads:

\[
\mathcal{D}(w) = j\omega w - \frac{m}{r} A_\theta^T w + \frac{1}{r} A_c^T w + \frac{1}{r} \frac{\partial (r A_r^T w)}{\partial r} + \frac{\partial (A_z^T w)}{\partial z}.
\]

The stabilisation parameter \(\tau_i\) within each element of the discretisation is the one used for instance by Rao and Morris [14]:

\[
\tau_i = \max (\alpha h_{i,l} / \lambda I), \quad \text{with } \alpha \text{ a stabilisation coefficient, } h_{i,l} \text{ the element size}
\]
of the $i^{th}$-element in the $l^{th}$-direction ($z$ or $r$). $\lambda_l$ is the spectral radius of the coefficient matrix $A_l$ and $I$ is the $5 \times 5$ identity matrix. In the numerical computations $\alpha = 1/(2p)$: the coefficient $1/2$ gives a value of $\tau_i$ analogous to the steady convective-diffusive equation, and the factor $1/p$ accounts for the use of high-order shape functions [12].

A linear system $Ks = f$ is obtained, where the complex-valued sparse matrix $K$ contains the different contributions of the physical integrals, $f$ is the source vector and $s$ is the solution vector. In the $p$-FEM, the space basis is enriched with edge and bubble shape functions. The Lobatto shape functions are used in the numerical simulations, because of their hierarchic property and good conditioning [9].

Specific boundary conditions are applied: an axial symmetry condition is enforced along the boundary at $r = 0$ and hard-wall conditions ($u' \cdot n = 0$) are specified at the engine duct walls. PML are applied to inject the modes inside the bypass duct and to absorb the outgoing waves at the far-field and in the core duct. More details on the application of the boundary conditions can be found in [17].

4. Numerical results

The numerical model has first been validated for a simple straight duct geometry [17]. On the basis of this validation, a more complex test case is studied. A realistic exhaust geometry is investigated, for two configurations: the no-flow condition with quiescent fluid, and the static-approach condition with non-uniform mean flow inside the duct and fluid at rest outside the duct, as shown in Fig. 1b. The test case has been developed within the European project TURNEX [18] and used by other authors to benchmark Computational Aero Acoustics (CAA) methods [15, 19].

Figure 1a shows the geometry of the exhaust with an unstructured mesh composed of linear triangular elements. The computational domain stretches for $z$ from $-0.493$ m to $1.27$ m and for $r$ from 0 to $0.65$ m, and is surrounded by a PML. The acoustic modes are injected through the PML at the inlet of the bypass duct: a duct mode $(m, n)$ of unit incident intensity propagates inside the duct. A PML is also added at the core duct inlet. In order to check the solution and its convergence, control points are located on the circle centred on the point $(0, 0)$ and of radius $R = 0.5$ m. The angle $\Phi$ defined along this circle is measured from the positive $z$-direction. The results obtained from the numerical simulations are discussed in the next sections.

![Figure 1: Mesh for the acoustic computation (a) and Mach number for the static-approach case (b).](image)

4.1 No-flow case

Let us first examine the results obtained in the no-flow configuration. The mesh is chosen as follows: a characteristic element size $h = 0.03$ m is applied in the majority of the domain, whereas the refinement is $h_{wall} = 0.02$ m along the duct walls and $h_{tip} = 0.005$ m around the ducts lips. The PML thicknesses are of the size of the mesh refinement in the corresponding region, namely $h_{wall}$ in the ducts PML and $h$ at the outlet of the domain. These dimensions give a mesh with 7,274 elements,
shown in Fig. [1a] The frequency of the propagating wave is $f = 8282$ Hz. The corresponding Helmholtz number is $k_0a = 19.3$, with $k_0 = 2\pi f / c_0$ and $a = 0.126$ m the outer radius of the bypass duct for the acoustic simulation. The speed of sound and the density are taken to be respectively $c_0 = 340.17$ m/s and $\rho_0 = 1.225$ kg/m$^3$.

The pressure fields obtained for the duct modes $(0, 1)$ and $(9, 1)$ are represented in Fig. [2]. For a better comparison the color scales are the same as the one used in the reference paper of Iob at al. [15]. The same main features are observed: the acoustic wave propagates inside the bypass duct and the main radiation directions are observed. For the mode $(0, 1)$ the directions of main intensity in the SPL directivity are located along the symmetry axis, and in the directions $\Phi \in [30^\circ; 50^\circ]$ from the bypass duct lip and $\Phi = 12^\circ$ from the core duct lip. For the mode $(9, 1)$ the main directivity is observed from the core duct lip for $\Phi \in [20^\circ; 45^\circ]$.

![Figure 2: Real part of the pressure field. No-flow case.](image)

(a) Mode $(0, 1)$. $\Re(p') \in [-26.5 \text{ Pa}; 26.5 \text{ Pa}]$.

(b) Mode $(9, 1)$. $\Re(p') \in [-23.5 \text{ Pa}; 23.5 \text{ Pa}]$.

In addition, the Sound Pressure Level (SPL) is measured along the reference circle previously defined. Figure [3] shows the SPL against the directivity angle $\Phi$, for $p$ varying from 2 to 6, for both the duct modes $(0, 1)$ and $(9, 1)$. A $p$-convergence is observed and, with the current mesh, the order $p = 3$ already gives a good approximation of the solution in the near field. The total number of degrees of freedom (DOF) is 155,075 for $p = 3$. For comparison, the standard FEM based on quadratic shape functions gives a good approximation of the solution for a refined mesh corresponding to a total number of 381,760 DOF.

![Figure 3: SPL along the reference circle ($p_{\text{ref}} = 2 \times 10^{-5}$Pa). No-flow case. $p$ from 2 to 6.](image)

(a) Mode $(0, 1)$.

(b) Mode $(9, 1)$.

**4.2 Static-approach case**

Let us now examine the results obtained in the static-approach configuration. The mean flow properties at the ducts inlets are: for the bypass duct, $M_{\text{bypass}} = 0.447$, $c_0 = 347.19$ m/s, $\rho_0 =$
1.177 kg/m³ and for the core duct, \( M_{\text{core}} = 0.223, c_0 = 527.62 \text{ m/s}, \rho_0 = 0.509 \text{ kg/m}^3 \). Far from the exhaust, the mean flow is taken to be \( M_\infty = 0 \). The Computational Fluid Dynamics (CFD) data and the derivatives of the mean flow velocity are linearly interpolated on the nodes of the acoustic mesh. Figure 1b shows the Mach number contours in the computational domain, obtained for a refined mesh \((h = 0.01 \text{ m}, h_{\text{wall}} = 0.001 \text{ m} \text{ and } h_{\text{lip}} = 0.0001 \text{ m})\). For the acoustic computation the mesh is the same as for the no-flow condition. The frequency of the duct mode injected in the bypass duct is \( f = 7497 \text{ Hz} \). The corresponding Helmholtz number is \( k_0 a = 17.5 \). The pressure fields of the modes \((0, 1)\) and \((9, 1)\) are shown in Fig. 4. The SPL are plotted in Fig. 5, for \( p \) varying from 2 to 6. The convergence is observed, and a good agreement is obtained already with \( p = 3 \). The features observed in paper [15] are also obtained. The shear layers develop at the continuation of the bypass and core ducts lips, as shown shown in Fig. 6, with a closeup view of the real parts of the momentum components.

![Image 1](image1.png)

(a) Mode \((0, 1)\). \( p' \in [-35.5 \text{ Pa}; 35.5 \text{ Pa}] \).

![Image 2](image2.png)

(b) Mode \((9, 1)\). \( p' \in [-26 \text{ Pa}; 26 \text{ Pa}] \).

Figure 4: Real part of the pressure field. Static-approach case. \( p = 6 \).

![Image 3](image3.png)

(a) Mode \((0, 1)\).

![Image 4](image4.png)

(b) Mode \((9, 1)\).

Figure 5: SPL along the reference circle \((p_{\text{ref}} = 2 \times 10^{-5} \text{ Pa})\). Static-approach case. \( p \) from 2 to 6.

These colormaps illustrate the complexity of the physics with the presence of multiple-scale wavelengths. The acoustic waves are dominant in most of the domain. But, the vorticity waves tend to prevail close to the trailing edge. An efficient model should be able to adjust the numerical effort to the local dominant wavelengths. It could be achieved by modifying the mesh size and/or the interpolation order of the shape functions within each element. This pleads in favor of the implementation of an \((h,p)\)-adaptive scheme to optimise the computational cost while guaranteeing that the solution has converged in all the regions of interest.
5. Conclusion

In this paper, an axisymmetric high-order FEM implementation of the LEE in the frequency domain is presented. The method is applied to the propagation of duct modes inside a realistic turbofan engine exhaust, with complex geometry and non-uniform mean flow. The sound propagation and radiation are accurately described, as well as the interaction between the acoustic waves and the hydrodynamic field resulting in the vorticity shedding from the duct lips.

Two axes of improvement are identified. Firstly, the geometry representation could be improved by the introduction of curved elements on the boundaries. This would allow to use larger elements in the vicinity of the duct wall. Then, the implementation of an \((h, p)\)-adaptive scheme would help to optimise the computational cost across the domain while assuring that the solution has reached convergence everywhere. This will be the subject of forthcoming studies.

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